

*Preliminary Version*  
Evidential Value in ANOVA-Regression Results in  
Scientific Integrity Studies

Chris A.J. Klaassen,  
Korteweg-de Vries Institute for Mathematics  
University of Amsterdam  
P.O. Box 94248, 1090 GE Amsterdam, The Netherlands  
email: c.a.j.klaassen@uva.nl

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**Abstract**

Some scientific publications are under suspicion of fabrication of data. Since humans are bad random number generators, there might be some evidential value in favor of fabrication in the statistical results as presented in such papers. In case of ANOVA-Regression studies we present the evidential value of the results of such a study in favor of the hypothesis of a dependence structure in the underlying data, which indicates fabrication, versus the hypothesis of independence, which is the ANOVA model assumption. Applications of this approach are also presented.

## 1 Introduction

Consider a publication based on an empirical study in which the data are analyzed by an ANOVA-Regression model, as presented in Section 3. Assume that the study is under suspicion of data fabrication and that the underlying data are not available. Based on just the publication itself it has to be decided if the suspicion is justified. To this end we implement an idea of Simonsohn (2012), which states that typically when data are fabricated, the model fit is *too good to be true*, and we apply the by now standard approach in Forensic Statistics, which is a Bayesian one, as described in Section 2. This approach yields the so-called evidential value of the publication in favor of the hypothesis of a dependence structure in the underlying data, which indicates fabrication, versus the hypothesis of independence, which is the ANOVA model assumption. In Section 4, in particular in Theorem 4.1 we describe how this evidential value may be computed, or at least be

bounded from below and from above. This Theorem is proved in the Appendix. We apply our approach to Förster and Denzler (2012) in Section 5. Some notes on the interpretation of evidential value are presented in Section 6. Our approach is similar to the one in Klaassen (2013).

## 2 Evidential Value in Scientific Integrity Studies

The hypothesis  $H_F$  of fabrication of data has been put forward about a scientific publication. The author claims the hypothesis  $H_I$  of integrity of the data holds. A Committee on Scientific Integrity has to decide in favor of  $H_F$  or  $H_I$ . In line with the so-called *Bayesian Paradigm of Forensic Statistics* the Committee has to construct a prior opinion about  $H_F$  and  $H_I$ , i.e. before studying the evidence  $E$ , namely the paper itself and other evidence. This prior opinion is formulated in terms of the prior odds in favor of the hypothesis of fabrication, namely

$$P(H_F) / P(H_I).$$

Subsequently experts have to determine the probability that  $E$  occurs under both the hypothesis  $H_I$  that the data have been collected in the scientifically right way and the hypothesis  $H_F$  that the data have been manipulated or fabricated. The ratio of these probabilities

$$P(E | H_F) / P(E | H_I)$$

is called the likelihood ratio. Multiplying the prior odds and the likelihood ratio the Committee obtains the so-called posterior odds in favor of the hypothesis of fabrication

$$P(H_F | E) / P(H_I | E),$$

i.e., the odds in favor of  $H_F$  after having seen the evidence. The Committee has to base its decision on these posterior odds. In summary, the *Bayesian Paradigm of Scientific Integrity Studies* reads as follows

$$\underbrace{\frac{P(H_F)}{P(H_I)}}_{\text{prior odds}} \underbrace{\frac{P(E | H_F)}{P(E | H_I)}}_{\text{likelihood ratio}} = \underbrace{\frac{P(H_F | E)}{P(H_I | E)}}_{\text{posterior odds}}. \quad (1)$$

Since the likelihood ratio in (1) may be interpreted as the weight that the evidence should have in the decision of the Committee, it is called the evidential value in favor of the hypothesis of fabrication (versus the hypothesis of integrity).

The evidence  $E$  is viewed here as a realization of a random mechanism, both under  $H_F$  and  $H_I$ . In case this random mechanism produces outcomes via probability density functions  $f(E|H_F)$  and  $f(E|H_I)$ , the probabilities in the likelihood ratio or evidential value are replaced by the corresponding probability density functions, resulting in

$$\underbrace{\frac{P(H_F)}{P(H_I)}}_{\text{prior odds}} \underbrace{\frac{f(E|H_F)}{f(E|H_I)}}_{\text{evidential value}} = \underbrace{\frac{P(H_F|E)}{P(H_I|E)}}_{\text{posterior odds}}. \quad (2)$$

### 3 Modelling Fabrication of Data Underlying a Specific Type of ANOVA-Regression Studies

In one-way Analysis of Variance the basic assumption is that all observations may be viewed as realizations of independent normally distributed random variables with means that depend on the values of some categorical covariate. Let this categorical covariate take three values only, and let the number of observations for each of the three cells be the same, namely  $n$ . The random variables denoting the observations are then

$$X_{ij} = \mu_i + \varepsilon_{ij}, \quad i = 1, 2, 3, j = 1, \dots, n. \quad (3)$$

The cell means  $\mu_i$  are unknown real numbers, and the measurement errors  $\varepsilon_{ij}$  are independent, normally distributed random variables with mean 0 and variance  $\sigma_i^2$ ,  $i = 1, 2, 3$ . Contrary to the standard assumption in ANOVA we assume here that the variances may depend on the covariate. In our ANOVA-regression model there exist constants  $\alpha$  and  $\beta$ , such that

$$\mu_i = \alpha + \beta i, \quad i = 1, 2, 3. \quad (4)$$

Actually, this is equivalent to the following restriction on the means  $\mu_i$

$$\mu_1 - 2\mu_2 + \mu_3 = 0. \quad (5)$$

If authors are fiddling around with data and are fabricating and falsifying data, they tend to underestimate the variation that the data should show due to the randomness within the model. Within the framework of the above ANOVA-regression case, we model this by introducing dependence between the normal random variables  $\varepsilon_{ij}$ , which represent the measurement errors. Actually, we assume that the measurement errors in any cell may have nonzero correlation to the corresponding measurement errors in the

other cells. More precisely formulated, we assume that the correlations between the random variables  $\varepsilon_{ij}$  no longer all vanish, but satisfy

$$\rho(\varepsilon_{1j}, \varepsilon_{2j}) = \rho_3, \quad \rho(\varepsilon_{1j}, \varepsilon_{3j}) = \rho_2, \quad \rho(\varepsilon_{2j}, \varepsilon_{3j}) = \rho_1, \quad j = 1, \dots, n, \quad (6)$$

with all other correlations still being equal to 0. Because we restrict attention to multivariate normal densities in the sequel, we exclude  $|\rho_i| = 1$ , so we assume  $-1 < \rho_i < 1$ ,  $i = 1, 2, 3$ . We note that under the standard assumptions of ANOVA  $\rho_i = 0$  holds. Furthermore, we note that within cells observations may be renumbered in order to get the structure (6). Nevertheless, we still assume (3) to hold and the measurement errors to be normally distributed with mean 0 and variance  $\sigma_i^2$ ,  $i = 1, 2, 3$ . Since the covariance matrix of the  $\varepsilon_{i1}$ s has to be positive semidefinite, the determinant

$$\begin{vmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_3 & \sigma_1\sigma_3\rho_2 \\ \sigma_1\sigma_2\rho_3 & \sigma_2^2 & \sigma_2\sigma_3\rho_1 \\ \sigma_1\sigma_3\rho_2 & \sigma_2\sigma_3\rho_1 & \sigma_3^2 \end{vmatrix} = \sigma_1^2\sigma_2^2\sigma_3^2 (1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3) \quad (7)$$

has to be nonnegative, and hence we have the side condition  $1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3 \geq 0$  on the  $\rho$ 's. Again, for technical reasons we prefer to work with multivariate normal densities and hence we shall assume  $1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3 > 0$ .

A way in which fabrication of measurement errors may take place is by copying some of them with an additional multiplication and addition or subtraction. This might be modelled as follows. Let  $U_j$ ,  $j = 1, \dots, n$ , and  $V_{ij}$ ,  $i = 1, 2, 3$ ,  $j = 1, \dots, n$ , be independent and identically distributed standard normal random variables. Independent of these, let the random indicators  $\Delta_{ij}$ ,  $i = 1, 2, 3$ ,  $j = 1, \dots, n$ , be independent Bernoulli random variables with  $P(\Delta_{1j} = 1) = \sqrt{\rho_2\rho_3/\rho_1}$ ,  $P(\Delta_{2j} = 1) = \sqrt{\rho_1\rho_3/\rho_2}$ ,  $P(\Delta_{3j} = 1) = \sqrt{\rho_1\rho_2/\rho_3}$ , and  $P(\Delta_{1j} = 0) = 1 - P(\Delta_{1j} = 1)$ , etc. Then

$$\varepsilon_{ij} = \sigma_i (\Delta_{ij}U_j + (1 - \Delta_{ij})V_{ij}), \quad i = 1, \dots, I, \quad j = 1, \dots, n, \quad (8)$$

satisfy (3). Note that we have  $\varepsilon_{1j}/\sigma_1 = \varepsilon_{2j}/\sigma_2 = U_j$  with probability  $\rho_3$  then, and since analogous relations hold for the two other combinations, the measurement errors satisfy (6).

Finally, we note that (6) is just one possible way to model dependence, and that the actual way in which fabrication has been implemented, might lead to quite different dependence structures. However, this model will come close to some types of fabrication and falsification.

## 4 Evidential Value for Fabrication of Data Underlying an ANOVA-Regression Study

Consider a study in a scientific research paper. The data underlying this study are analyzed by the one-way layout ANOVA model (3) of the preceding

section and as results the sample means and sample standard deviations of the three cells are presented. The underlying data themselves are not published and are not available. According to the theory as developed in the research paper the linear regression condition (5) holds.

There are two hypotheses to be formulated about the data underlying this ANOVA-regression study. The hypothesis  $H_F$  of fabrication of the data underlying the results presented in the paper, is that  $\rho_i \neq 0$  holds for at least one  $i$ ,  $i = 1, 2, 3$ . The other hypothesis  $H_I$  represents the situation that data have been collected according to (3) with independent  $X_{ij}$ , i.e.,  $\rho_1 = \rho_2 = \rho_3 = 0$ . We want to determine the evidential value of the published results of the ANOVA-regression study, i.e., of the sample means and sample variances for the three cells, in favor of the hypothesis  $H_F$  versus  $H_I$ .

To this end we first note that the sample means in the cells,

$$X_i = \frac{1}{n} \sum_{j=1}^n X_{ij}, \quad i = 1, 2, 3, \quad (9)$$

have a joint trivariate normal distribution. Actually, the dependence structure (6) implies

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, n^{-1} \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_3 & \sigma_1\sigma_3\rho_2 \\ \sigma_1\sigma_2\rho_3 & \sigma_2^2 & \sigma_2\sigma_3\rho_1 \\ \sigma_1\sigma_3\rho_2 & \sigma_2\sigma_3\rho_1 & \sigma_3^2 \end{pmatrix} \right). \quad (10)$$

In stead of assuming normally distributed errors satisfying (6), we could have started right away from (10). This is a much weaker condition that in practice is more likely to be satisfied approximately in view of the central limit theorem.

By  $X$  we denote the column 3-vector with components  $X_1, X_2$ , and  $X_3$ . Let these components be uncorrelated and let  $A$  be a nonsingular  $3 \times 3$ -matrix such that the components  $Y_1, Y_2$ , and  $Y_3$  of  $Y = AX$  are uncorrelated as well, and hence by the normality assumption independent. The first row of  $A$  is chosen to be  $(1, -2, 1)$ , which entails  $Y_1 = X_1 - 2X_2 + X_3$ . The two other rows depend on the values of the parameters in the covariance matrix, but not on the value of the 3-vector  $\mu = (\mu_1, \mu_2, \mu_3)^T$ . So, for inference about  $\mu$  the vector  $Y$  is equivalent to  $X$ . Let  $\nu = (\nu_1, \nu_2, \nu_3)^T = EY$  be the expectation of  $Y$ . Because of the nonsingularity of  $A$  there does not exist a linear combination of  $\nu_2$  and  $\nu_3$  that equals  $\nu_1$ . By the independence of the components of  $Y$  this implies that the first component  $Y_1$  is a sufficient statistic for its expectation  $\nu_1 = \mu_1 - 2\mu_2 + \mu_3$ , which according to the theory as claimed by the paper under study vanishes, as in (5). This means that all information about  $\nu_1$  contained in the independent sample means  $X_1, X_2$ , and  $X_3$ , is contained in  $Y_1 = X_1 - 2X_2 + X_3$ . Therefore we will base our evidential value on this statistic, which we will rename as  $Z = X_1 - 2X_2 + X_3$ .

First we note that under the linear regression assumption (5) we have

$$\begin{aligned}\sqrt{n}(X_1 - 2X_2 + X_3) &= \sqrt{n}Z \sim \mathcal{N}(0, \sigma_Z^2), \\ \sigma_Z^2 &= \sigma_1^2 + 4\sigma_2^2 + \sigma_3^2 - 4\sigma_1\sigma_2\rho_3 + 2\sigma_1\sigma_3\rho_2 - 4\sigma_2\sigma_3\rho_1.\end{aligned}\quad (11)$$

This normal distribution depends on the parameters  $\rho_1, \rho_2, \rho_3, \sigma_1^2, \sigma_2^2, \sigma_3^2$ , with  $-1 < \rho_i < 1$ ,  $1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3 > 0$ ,  $0 < \sigma_i$ ,  $i = 1, 2, 3$ . In the studies we consider, only realizations  $x_i$  of the cell means  $X_i$  and estimates  $s_i^2$  of the cell variances  $\sigma_i^2$  are given,  $i = 1, 2, 3$ .

Let us denote the density of  $Z = X_1 - 2X_2 + X_3$  at  $z = x_1 - 2x_2 + x_3$  with  $\sigma_i$  replaced by  $s_i > 0$  by

$$\begin{aligned}f_n(z; \rho_1, \rho_2, \rho_3) &= \sqrt{\frac{n}{2\pi}} \frac{1}{s(\rho_1, \rho_2, \rho_3)} \exp\left(-\frac{nz^2}{2s^2(\rho_1, \rho_2, \rho_3)}\right), \\ s(\rho_1, \rho_2, \rho_3) &= (s_1^2 + 4s_2^2 + s_3^2 - 4s_1s_2\rho_3 + 2s_1s_3\rho_2 - 4s_2s_3\rho_1)^{1/2}.\end{aligned}\quad (12)$$

We will base our evidential value on this density, viewing  $s_1, s_2$ , and  $s_3$  as given.

The hypothesis  $H_I$  of proper data corresponds to  $\rho_1 = \rho_2 = \rho_3 = 0$ . We shall let the hypothesis  $H_F$  of fabrication of the data correspond to nonzero correlation between at least two sample means, such that

$$s(\rho_1, \rho_2, \rho_3) \leq s(0, 0, 0) \quad (13)$$

holds. This means that we restrict  $H_F$  by the condition that  $\sqrt{n}Z = \sqrt{n}(X_1 - 2X_2 + X_3)$  has a(n estimated) variance that equals at most the (estimated) variance under independence,  $H_I$ . This restriction is in line with our presumption that people when fabricating data tend to underestimate variation. The evidential value

$$\frac{f(E|H_F)}{f(E|H_I)}$$

from (2) in favor of  $H_F$  versus  $H_I$  becomes in this case (cf. Zhang (2009), Bickel (2012))

$$\mathbb{V} = \frac{\sup_{-1 < \rho_i < 1, 1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3 > 0, s(\rho_1, \rho_2, \rho_3) \leq s(0, 0, 0)} f_n(z; \rho_1, \rho_2, \rho_3)}{f_n(z; 0, 0, 0)}.\quad (14)$$

This evidential value may be computed with the help of the following Theorem.

**Theorem 4.1.** *Within the general model (11) and with the notation (12), define*

$$s_L^2 = \inf_{-1 < \rho_i < 1, 1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3 > 0, s(\rho_1, \rho_2, \rho_3) \leq s(0, 0, 0)} s^2(\rho_1, \rho_2, \rho_3)$$

and

$$\tilde{s}_L^2 = \min \left\{ (2s_2 - (s_1 + s_3))^2, \left( 2s_2 - \sqrt{s_1^2 + s_3^2} \right)^2 \right\},$$

and write  $s_0^2 = s^2(0, 0, 0) = s_1^2 + 4s_2^2 + s_3^2$ . Then

$$s_L^2 \leq \tilde{s}_L^2 \leq s_0^2 \quad (15)$$

holds. Furthermore, we have:

- If

$$\tilde{s}_L^2 \leq nz^2 \leq s_0^2$$

holds, then the evidential value from (14) becomes

$$\mathbb{V} = \frac{s_0}{\sqrt{nz^2}} \exp \left\{ -\frac{1}{2}nz^2 \left[ \frac{1}{nz^2} - \frac{1}{s_0^2} \right] \right\} \geq 1. \quad (16)$$

- If

$$nz^2 \leq \tilde{s}_L^2$$

holds, then the evidential value from (14) satisfies

$$\mathbb{V} \geq \frac{s_0}{\tilde{s}_L} \exp \left\{ -\frac{1}{2}nz^2 \left[ \frac{1}{\tilde{s}_L^2} - \frac{1}{s_0^2} \right] \right\} \geq 1 \quad (17)$$

and equals at most the left hand side of inequality (16).

- If

$$s_0^2 \leq nz^2$$

holds, then the evidential value from (14) becomes  $\mathbb{V} = 1$ .

The proof is given in the Appendix.

## 5 Application

A complaint has been filed about the scientific integrity of Förster and Denzler (2012). This paper contains 12 studies modelled as in Section 3. The three cell means,  $x_1, x_2, x_3$ , for each study have been given in the paper. The sample standard deviations for these cells,  $s_1, s_2, s_3$ , have been provided by the authors to the investigator who filed the complaint. These data and the corresponding evidential values are given in Table 1.

To interpret these evidential values it is useful to consider also the evidential values that are obtained for similar publications in the same field as collected in the complaint; see Table 2.

We notice that these evidential values from literature are all below 2, say, except for Lerouge-2, Malkoc, and Smith-4. In contrast all evidential

Table 1: The evidential values of 12 studies from Förster and Denzler (2012).

Study	$n$	$x_1, x_2, x_3$	$s_1, s_2, s_3$	$\mathbb{V}$
1	20	2.47, 3.04, 3.68	1.21, 0.72, 0.68	3.92
2	20	2.51, 2.95, 3.35	0.71, 0.49, 0.64	4.68
3	20	2.40, 2.90, 3.45	0.86, 0.51, 0.80	4.26
4	20	2.41, 2.98, 3.64	1.07, 0.51, 0.95	2.72
5	20	2.14, 2.82, 3.41	1.20, 0.78, 0.71	3.21
6	20	3.19, 4.01, 4.79	1.07, 1.21, 0.82	4.95–9.41
7	20	2.63, 3.73, 4.73	1.49, 1.21, 1.55	4.43
8	20	2.87, 3.83, 4.79	1.24, 1.09, 1.53	13.95– $\infty$
9a	20	2.35, 3.66, 4.76	1.01, 1.19, 1.71	2.10
9b	15	2.55, 3.72, 4.78	1.16, 1.00, 1.47	3.95
10a	20	2.66, 3.69, 4.81	1.21, 1.30, 1.54	4.94
10b	15	2.42, 3.73, 5.02	0.82, 1.28, 1.45	10.17–23.92

Table 2: Evidential values of 21 studies from the social psychology literature.

Study	$n$	$x_1, x_2, x_3$	$s_1, s_2, s_3$	$\mathbb{V}$
Hagtvedt-1	141/6	4.39, 3.97, 3.84	0.76, 1.26, 1.14	1.40
Hagtvedt-2	141/6	3.22, 3.84, 4.11	0.98, 1.02, 1.46	1.17
Hunt	75/3	1.48, 1.04, 1.04	0.82, 0.68, 0.68	1
Jia	132/3	1.09, 0.70, 0.59	0.89, 0.69, 0.62	1
Kanten-1	269/6	3.29, 3.14, 2.66	1.11, 0.94, 0.71	1.001
Kanten-2	269/6	3.02, 2.99, 2.85	0.80, 0.84, 0.70	1.75
Lerouge-1	63/3	4.24, 2.48, 2.14	1.51, 2.16, 2.13	1
Lerouge-2	63/3	2.95, 2.81, 2.62	2.44, 1.81, 2.25	12.23–13.01
Lerouge-3	54/3	4.90, 3.31, 2.79	2.22, 2.09, 1.66	1.01
Lerouge-4	54/3	3.69, 2.67, 2.50	2.78, 2.51, 1.66	1.21
Malkoc	521/3	4.72, 5.36, 6.19	4.96, 9.08, 10.58	5.26–5.27
Polman	65/3	4.69, 3.50, 2.91	2.37, 2.09, 2.42	1.34
Rook-1	168/6	6.22, 6.13, 4.73	3.05, 2.19, 1.95	1
Rook-2	168/6	5.39, 5.22, 4.61	2.14, 2.58, 2.28	1.69
Smith-1	73/3	4.38, 4.26, 3.55	1.53, 1.36, 1.07	1.01
Smith-2	76/3	14.83, 12.69, 11.88	4.62, 4.95, 4.75	1.26
Smith-3	113/3	0.42, 0.53, 0.56	0.20, 0.19, 0.19	1
Smith-4	140/3	4.70, 7.90, 11.80	7.40, 11.40, 20.40	4.04
Smith-5	125/3	14.52, 13.43, 12.85	2.81, 3.27, 3.94	1.63
Smith-6	97/3	10.85, 8.64, 8.32	5.07, 3.61, 4.17	1
Smith-7	144/3	4.64, 4.84, 5.49	1.30, 1.56, 1.28	1.02



values from Förster and Denzler (2012) are above 2. From Table 2 one might estimate the probability  $P_{H_I}(\mathbb{V} \geq 2)$  under the hypothesis  $H_I$  that the evidential value will equal at least 2, as  $3/21 = 1/7$ . This would imply that the probability that 12 studies will have an evidential value of at least 2, as occurs in Table 1, equals approximately  $(1/7)^{12} \approx 7.2 \times 10^{-11}$ .

On the other hand, Theorem 4.1 shows that  $\mathbb{V} \geq v > 1$  implies

$$\frac{s_0}{\sqrt{nz^2}} \exp \left\{ \frac{1}{2} \left[ \frac{nz^2}{s_0^2} - 1 \right] \right\} \geq v, \quad \frac{nz^2}{s_0^2} \leq 1. \quad (18)$$

For  $v = 2$  some computation shows that this means approximately

$$-0.3191 \leq \frac{\sqrt{n}(x_1 - 2x_2 + x_3)}{\sqrt{s_1^2 + 4s_2^2 + s_3^2}} \leq 0.3191. \quad (19)$$

Viewing the sample means  $x_i$  and sample variances  $s_i^2$  as random, we see that the ratio in (19) has approximately a standard normal distribution under the hypothesis  $H_I$ , which implies  $P_{H_I}(\mathbb{V} \geq 2) \approx 0.2504$ . This is much more than the  $1/7$  estimated from Table 2, but it still shows that the probability that 12 studies will have an evidential value of at least 2, as in Table 1, equals approximately  $(0.2504)^{12} \approx 6.1 \times 10^{-8}$ .

## 6 Interpreting Evidential Value

With the evidential value  $\mathbb{V}$  defined as in (14) through (17) the Bayesian paradigm for criminal court cases (2) becomes

$$\underbrace{\frac{P(H_F)}{P(H_I)}}_{\text{prior odds}} \underbrace{\mathbb{V}}_{\text{evidential value}} = \underbrace{\frac{P(H_F | E)}{P(H_I | E)}}_{\text{posterior odds}}. \quad (20)$$

An important principle in criminal court cases is ‘in dubio pro reo’, which means that in case of doubt the accused is favored. In science one might argue that the leading principle should be ‘in dubio pro scientia’, which should mean that in case of doubt a publication should be withdrawn. Within the framework of this paper this would imply that if the posterior odds in favor of hypothesis  $H_F$  of fabrication equal at least 1, then the conclusion should be that  $H_F$  is true. So an ANOVA-regression study for which

$$\underbrace{\frac{P(H_F)}{P(H_I)}}_{\text{prior odds}} \underbrace{\mathbb{V}}_{\text{evidential value}} = \underbrace{\frac{P(H_F | E)}{P(H_I | E)}}_{\text{posterior odds}} > 1 \quad (21)$$

holds, should be rejected and disqualified scientifically. Keeping this in mind one wonders what a reasonable choice of the prior odds would be.

In criminal court cases the choice of prior odds is left to the judge, and the evidential value has to be determined by the forensic expert.

We conclude with some notes.

- ANOVA studies are based on the assumption of normality. Often this assumption is not satisfied, but the technique is still applied. In view of the central limit theorem cell means like in our basic model (10) behave approximately like (jointly multivariate) normal random variables.
- Note that Theorem 4.1 implies

$$\mathbb{V} \geq 1.$$

Consequently, within this framework there does not exist exculpatory evidence. This is reasonable since bad science cannot be compensated by very good science. It should be very good anyway.

- When a paper contains more than one study based on independent data, then the evidential values of these studies can and may be combined into an overall evidential value by multiplication in order to determine the validity of the whole paper.
- We have modelled the hypothesis of data fabrication via (10). However, other, nonnormal multivariate distributions might model fabrication better. Consequently, higher evidential values might be possible.
- The discussion at the end of Section 5, in particular the argument involving (18) and (19), shows that the approach of evidential value  $\mathbb{V}$  is just a way to interpret the value of the statistic

$$Z_{\mathbb{V}} = \frac{\sqrt{n}(X_1 - 2X_2 + X_3)}{\sqrt{S_1^2 + 4S_2^2 + S_3^2}}, \quad (22)$$

where  $S_i^2$  is the sample variance viewed as a random variable. Note that  $Z_{\mathbb{V}}$  has a standard normal distribution approximately. When it takes on a very small (absolute) value or small (absolute) values repeatedly, the suggestion is raised that data have been manipulated; note that if (5) does not hold,  $|Z_{\mathbb{V}}|$  will take on large values for  $n$  large; see the note below. The investigator who filed the complaint, based his argument on the ANOVA framework, tested the hypothesis (5) by rejecting it for large (absolute) values of the statistic

$$Z_C = \frac{\sqrt{n}(X_1 - 2X_2 + X_3)}{\sqrt{2(S_1^2 + S_2^2 + S_3^2)}}, \quad (23)$$

and noted that the  $p$ -values corresponding to the studies from Table 1 are suspiciously close to 1, in contrast to those from Table 2.

Note that both  $Z_V$  and  $Z_C$  have a normal distribution asymptotically as  $n \rightarrow \infty$  under (5) with mean 0 and variance 1 and  $(\sigma_1^2 + 4\sigma_2^2 + \sigma_3^2)/[2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)] \in (1/2, 2)$ , respectively. Under  $\sigma_1 = \sigma_2 = \sigma_3$  they are both standard normal asymptotically.

- By asymptotic theory  $Z_V$  from (22) has a normal distribution approximately, with mean  $\sqrt{n}(\mu_1 - 2\mu_2 + \mu_3)/\sqrt{\sigma_1^2 + 4\sigma_2^2 + \sigma_3^2}$  and variance 1. Since normal densities are unimodal and symmetric around their mean, this implies that  $P(-v \leq Z_V \leq v)$ ,  $v > 0$ , attains its maximum value under (5), at least approximately. This observation supports the heuristic that  $Z_V$  discerns between  $\mu_1 - 2\mu_2 + \mu_3 = 0$  and  $\mu_1 - 2\mu_2 + \mu_3 \neq 0$ . A similar observation holds for  $Z_C$ .
- Since  $Z_V$  and  $Z_C$  are quite similar statistics, the difference between the approach in the present paper and the approach of the compliant is basically the difference between a Bayesian and a frequentist approach. These are just two methods to interpret the data and they point in the same direction, typically.

## A Appendix: Proof

Here we present a proof of Theorem 4.1. In view of

$$s^2(1, 1, 1) = s_0^2 - 4s_1s_2 + 2s_1s_3 - 4s_2s_3 = (2s_2 - s_1 - s_3)^2, \quad (24)$$

$$s^2\left(\frac{s_3}{\sqrt{s_1^2 + s_3^2}}, 0, \frac{s_1}{\sqrt{s_1^2 + s_3^2}}\right) = s_0^2 - 4s_2\sqrt{s_1^2 + s_3^2} = \left(2s_2 - \sqrt{s_1^2 + s_3^2}\right)^2$$

we arrive at the inequalities of (15). The proof of the theorem is completed by repeated application of the following lemma.

**Lemma A.1.** *The function*

$$x \mapsto \frac{1}{\sqrt{x}} e^{-\lambda/x}$$

*is increasing from 0 at 0 to  $1/\sqrt{2e\lambda}$  at  $x = 2\lambda$ , and subsequently decreasing to 0 at  $\infty$ . Furthermore, the function*

$$x \mapsto \frac{1}{\sqrt{x}} e^{\frac{1}{2}(x-1)}$$

*attains its minimum value 1 on  $(0, \infty)$  at  $x = 1$ .*

**Proof**

Differentiation yields these results. □

## References

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